important part in the physical theory are satisfied by $F$ as soon as we subject $f$ to the Maxwell equations.

[1] Cf. a note by the present writer, *Paris, C. R. Acad. Sci.*, **176**, pp. 1294–1297.

[2] These PROCEEDINGS, **9**, No. 6, June, 1923 (179–183).

[3] This will be presented in detail elsewhere.

[4] Indications as to how matter can be introduced in this theory were given in a paper read before the *Amer. Math. Soc.;* see their *Bull.*, **1923**, p. 148.

[5] *Edinburgh, Proc. R. Soc.*, **42** (1–23).

[6] Formula (11) of the paper referred to under 2.

[7] Cf. Kottler, F., *Wien. Sitz Ber. Ak. Wiss. IIa.*, **131**, pp. 119–146.

[8] These PROCEEDINGS, **9**, 1923 (401–403).

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## ON THE DEFORMATION OF AN *n*-CELL

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By an extremely simple device, Professor Veblen (These PROCEEDINGS, **3,** 1917, p. 655) has proved that every one-one continuous transformation of an $n$-cell and its boundary into themselves which leaves invariant all points of the boundary may be realized by a deformation. However, the deformation is of such a character that the boundary of the $n$-cell does not remain invariant but merely returns to its initial position as the deformation is completed. A slight modification of Professor Veblen's scheme gives a deformation during the entire course of which the boundary remains pointwise invariant. I shall give the proof only for the case $n = 2$, since the generalization to higher dimensions is immediate.

Let the 2-cell and its boundary be represented by the interior and periphery of the unit circle $C_1$. Any one-one continuous transformation $T$ of the 2-cell into itself may then be expressed by a pair of equations in polar coördinates of the following form:

$$r^1 = R\,(r, \varphi), \qquad \varphi^1 = \Theta\,(r, \varphi), \tag{T}$$

where $R$ and $\Theta$ are defined within the unit circle. Moreover, if the transformation is pointwise invariant on the boundary $C_1$ of the 2-cell, we must also have

$$R\,(1, \varphi) = 1, \qquad \Theta\,(1, \varphi) = \varphi.$$

We shall extend the region of definition of the transformation $T$ over the entire plane by putting

$$R\,(r, \varphi) = r,\ \Theta\,(r, \varphi) = \varphi, \text{ for } (r \geqq 1).$$

Clearly, then, the extended transformation will leave invariant all points on or without the unit circle $C_1$.

Now, consider the family of transformations

$$r^1 = \lambda R(^r/_\lambda, \varphi), \qquad \varphi^1 = \Theta(^r/_\lambda, \varphi), \qquad (1 \geq \lambda > 0) \qquad (T_\lambda)$$

where $\lambda$ is a parameter. Evidently, the transformation $T_\lambda$ leaves invariant all points on or without a circle $C_\lambda$ of radius $\lambda$ with centre at the origin, and determines within the circle $C_\lambda$ a transformation which is merely a replica on a different scale of the transformation determined by $T$ within the unit circle $C_1$. Thus, as $\lambda$ approaches zero, the transformation $T_\lambda$ merges continuously into the identity, which makes it reasonable to define $T_0$ as the identity. For $\lambda = 1$, the transformation $T_\lambda$ reduces to $T$.

The desired deformation is the one determined by $T_\lambda$ as $\lambda$ increases continuously from 0 to 1. Evidently, the unit circle remains invariant during the deformation, since it is never interior to the circle $C_\lambda$.

The above theorem was proved for the case $n = 2$ by H. Tietze (*Palermo, Rend., Circ. Mat.*, **38,** 1914, pp. 247–304) and, more simply, by H. L. Smith (*Annals Math.*, **19,** 1918–19, pp. 137–141). The present proof, however, generalizes at once to $n$ dimensions, as has already been remarked.

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# *CONTROL OF THE APPEARANCE OF PUPA-LARVAE IN PAEDOGENETIC DIPTERA*

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It has been known for some time (Wagner 1861) that Miastor reproduces during much of its life-cycle by paedogenesis. Ordinarily this mode of reproduction maintains throughout the autumn, winter and spring, until early summer when pupae occur, giving rise to male and female imagines. With these two modes of reproduction occurring, viz. paedogenesis and normal adult sexual reproduction, it is natural that there should be polymorphism among the larvae of Miastor,[1] one type reproducing by paedogenesis and incapable of metamorphosing into pupae, the other type giving rise to pupae but incapable of paedogenesis.

It seemed desirable to ascertain, if possible, what factors control the appearance of the different types, the more so as I had observed, in one instance, pupa-larvae and pupae occurring in nature together with paedo-